A-Level
Mathematics

MM04 Mechanics 4
Final Mark scheme

6360
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Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

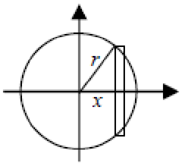
Q1	Solution	Mark	Total	Comment
	Use of md^2 $d^2 + 1.5(2d)^2 + 2(5d)^2$	**M1** **A1,A1**		Correct use at least once **A1** first two terms, **A1** final term
	Hence $57d^2 = 513$	**m1**		Forming their correct equation dependent on first **M1**
	$d = 3$	**A1**	5	**CAO**
	Total		**5**	

Q2	Solution	Mark	Total	Comment
(a)	Moments about E $Wl + W(1.5l) = 2lY$	**M1A1**		**M1** one term correct, **A1** fully correct
	$Y = \dfrac{5W}{4}$	**A1**	3	**CSO** – Printed answer
(b)	Resolving forces at C vertically $T_{BC} \cos 30^0 = Y$	**M1**		Fully correct equation involving T_{BC}
	$T_{BC} = \dfrac{5\sqrt{3}W}{6}$ (BC is in compression)	**A1**		Any equivalent form
	Resolve forces at B vertically $T_{BC}\cos 30^0 = W \pm T_{BD}\cos 30^0$	**M1**		**M1 -** Forms equation involving T_{BD} and T_{BC} must realise that rod BC is in compression
	$T_{BC}\cos 30^0 = W + T_{BD}\cos 30^0$	**A1F**		Fully correct equation and substitutes **their** T_{BC} correctly
	$T_{BD} = \dfrac{\sqrt{3}W}{6}$	**A1**	5	**CSO**
(c)	Rod AB is in compression.	**E1**		**CAO** - Clearly identifies that rod AB is in compression.
	Rod BD is in tension and rod BC is in compression hence rod AB must be in compression to ensure that the horizontal forces are in equilibrium at point B.	**E1**	2	Valid reason given – must refer to equilibrium of forces.
	Total		**10**	

Q3	Solution	Mark	Total	Comment
(a)	Volume = $\pi\int_0^{9h}\frac{x^2}{9}dx$			
	$\left[\frac{x^3}{27}\right]_0^{9h}\pi = 27h^3\pi$	**B1**		Correct answer – volume formula can be quoted but must be used correctly
	$\pi\int_0^{9h}\frac{x^3}{9}dx = \left[\frac{x^4}{36}\right]_0^{9h}\pi = \frac{729}{4}h^4\pi$	**M1A1**		**M1** fully correct integration – **A1** correct evaluation with correct limits
	$\bar{x} = \dfrac{\frac{729}{4}h^4\pi}{27h^3\pi} = \frac{27h}{4}$	**m1** **A1**	5	**m1** Divides expressions - dependent on **M1** above **A1** obtains correct answer
(b)	$\tan\theta = \dfrac{3h}{9h/4}$	**M1** **A1**		Use of tangent seen Correct distances used on RHS
	$\theta = 53^0$	**A1**	3	**CAO** - Correct angle found.
	Total		**8**	

Q4	Solution	Mark	Total	Comment
	MI$_{\text{ROD}}$ = $\frac{4}{3}(2m)(\frac{l}{2})^2 = \frac{2}{3}ml^2$	**B1**		**CAO** - Can be unsimplified
	MI$_{\text{PARTICLE+ROD}}$ = $\frac{2}{3}ml^2 + md^2$	**B1**		**CAO** - Combined MI for particle and rod
	Conservation of angular momentum $\frac{2}{3}ml^2\omega = (\frac{2}{3}ml^2 + md^2)(\frac{2}{3}\omega)$	**M1** **A1F**		**M1** - Forming an equation with one term correct **A1F** for equation fully correct Follow through errors from MI
	$d = \frac{l}{\sqrt{3}}$	**A1**	5	**CSO**
	Total		**5**	

Q5	Solution	Mark	Total	Comment
	$\begin{pmatrix}2p\\1\\0\end{pmatrix}\times\begin{pmatrix}2p\\1\\2\end{pmatrix}=\begin{pmatrix}2\\-4p\\0\end{pmatrix}$	**M1**		Correct evaluation of **rxF** or **Fxr** at least once
	$\begin{pmatrix}1\\2\\0\end{pmatrix}\times\begin{pmatrix}p\\-2\\3\end{pmatrix}=\begin{pmatrix}6\\-3\\-2-2p\end{pmatrix}$	**A1**		Two correct evaluations of **rxF** or **Fxr**
	$\begin{pmatrix}-2\\-1\\2\end{pmatrix}\times\begin{pmatrix}2\\-p\\2p\end{pmatrix}=\begin{pmatrix}0\\4p+4\\2p+2\end{pmatrix}$	**A1**		Three fully correct evaluations of **rxF** or **Fxr**
	Total $=\begin{pmatrix}8\\1\\0\end{pmatrix}$	**m1**		Totalling **their** three vectors above
		A1		**CAO**
	No p hence independent of p	**E1**		Final comment made about p
			6	
	Total		**6**	

Q6	Solution	Mark	Total	Comment
(a)	 Let density = ρ $m=\frac{4}{3}\pi r^3\rho \Rightarrow \rho=\frac{3m}{4\pi r^3}$	B1		p and m linked anywhere
	Mass of elemental disc / cylinder $=\pi\left(r^2-x^2\right)\delta x\rho$	M1		Use of volume of cylinder
	MI of elemental disc = $\frac{1}{2}\left[\pi\left(r^2-x^2\right)\delta x\rho\right]\left[r^2-x^2\right]$	A1		Correct formation of MI for disc
	MI of sphere = $\int_{-r}^{r}\frac{1}{2}\pi\rho\left(r^2-x^2\right)^2\,\mathrm{d}x$ $=\frac{3m}{8r^3}\int_{-r}^{r}\left(r^4+x^4-2r^2x^2\right)\,\mathrm{d}x$	m1		Evaluates integral - dependent on first **M1**
	$=\frac{3m}{8r^3}\left[r^4x+\frac{x^5}{5}-2r^2\frac{x^3}{3}\right]_{-r}^{r}$ $=\frac{3m}{8r^3}\left[2\left(r^5+\frac{r^5}{5}-\frac{2r^5}{3}\right)\right]$	A1F		**A1F** - Correct integration of **their** expression but must have correct number of terms
	$=\frac{3m}{4r^3}\times\frac{8r^5}{15}=\frac{2}{5}mr^2$	A1	6	**CSO** - Printed answer
(b)(i)	MI = $\frac{2}{5}mr^2+m(3r)^2$	M1		Correct structure for parallel axis theorem
	$\frac{2}{5}mr^2+9mr^2$ $\frac{47}{5}mr^2$	A1	2	**CAO** - Printed answer
(ii)	$\text{MI}_{\text{ROD}}=\frac{(2m)(2r)^2}{3}=\frac{8mr^2}{3}$	B1		Correct MI for rod – can be unsimplified
	$\text{MI}_{\text{LARGE SPHERE}}=$ $\frac{2}{5}(4m)(2r)^2+(4m)(4r)^2=\frac{352}{5}mr^2$	M1 A1		Correct structure for parallel axis theorem MI of large sphere correctly found
	Total = $\frac{352}{5}mr^2+\frac{8mr^2}{3}+\frac{47}{5}mr^2=\frac{1237}{15}mr^2$	M1 A1	5	Totalling their three MI **CAO**

(iii)	$\frac{1}{2}(\frac{1237mr^2}{15})\omega^2$	**M1** **A1F**		**M1** - Use of formula for rotational kinetic energy **A1F** - **Their** correct expression – follow through their previous answer
	Net Loss in PE = $16mgr - 3mgr$ $13mgr$	**M1** **A1**		**M1** - Use of formula for potential energy **A1** - Correct 'net' loss obtained
	$\frac{1}{2}(\frac{1237mr^2}{15})\omega^2 = 13mgr$	**M1**		Forming an equation using conservation of total PE/KE
	$\omega = \sqrt{\frac{390g}{1237r}}$	**A1**	6	**CAO**
	Total		**19**	

Q7	Solution	Mark	Total	Comment
(a)	$\mathbf{F}_1 = \begin{pmatrix} 3a \\ -a \end{pmatrix}$ Since the direction vector of the line of action of $\mathbf{F}_1$ represents the ratio of the force components	**B1**	1	Or equivalent statement

(b)	$\mathbf{F}_2 = \begin{pmatrix} -b \\ 2b \end{pmatrix}$	**B1**		Correct form for $\mathbf{F_2}$
	$3a - b = k$ $-a + 2b = 0$	**M1** **A1**		Forms two equations using horizontal and vertical components Both equations correct
	Moments about O $4(2b) - 2(a) - 5(3a) = -39$ Simplifies to $8b - 17a = -39$	**M1** **A1** **A1**		**M1** One correct pairing (force x distance) **A1** All signs consistent **A1** fully correct equation – can be unsimplified
	$a = 3$, $b = 1.5$ and $k = 7.5$	**M1** **A1** **A1**		**M1** solving **their** system of three equations to find any one unknown **A**1 for each other value - **CAO**
	$\mathbf{F}_1 = \begin{pmatrix} 9 \\ -3 \end{pmatrix}$ and $\mathbf{F}_2 = \begin{pmatrix} -1.5 \\ 3 \end{pmatrix}$	**M1** **A1F**	 11	**M1** - Substitutes **their** values to find both forces **A1F** – **Their** forces correctly found
	Total		**12**	

Q8	Solution	Mark	Total	Comment
(a)	Conservation of energy $\frac{1}{2}(24ma^2)\dot{\theta}^2 = mg(3a - 3\sqrt{2}a\cos\theta)$	**M1** **A1A1**		**M1** - Use of KE **and** PE **A1** each side
	$\dot{\theta}^2 = \frac{g}{4a}(1 - \sqrt{2}\cos\theta)$	**A1**	**4**	Printed answer
(b)	$2\dot{\theta}\ddot{\theta} = \frac{g\sqrt{2}\sin\theta}{4a}\dot{\theta}$	**M1**		Differentiating to find angular acceleration
	$\ddot{\theta} = \frac{g\sqrt{2}\sin\theta}{8a}$	**A1**		**CAO**
	$mg\sin\theta - Y = mr\ddot{\theta}$	**M1**		Correct structure used for Newton's Second Law
	$Y = mg\sin\theta - \frac{3\sqrt{2}amg\sqrt{2}\sin\theta}{8a}$	**A1** **A1F**		**A1** correct r, **A1**F substituting their angular acceleration
	$Y = \frac{mg\sin\theta}{4}$	**A1**	**6**	Fully simplified

	Total		**10**	